Optimal Fusion Algorithm Based on Multi-Sensor Tracking

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Abstract - *An optimal fusion algorithm for tracking maneuvering target based on centralized structure of multi-sensor is proposed. This algorithm is implemented with two filters and fuzzy logic using state fusion, together with the current statistic model and adaptive filtering. Firstly, the optimal weighting coefficients are obtained using the stochastic approximation theory, a suitable method of estimation measurements noise variance is developed based on fuzzy inference. Secondly, two adaptive Unscented Kalman filters with current statistical model are derived in parallel, and fuzzy rule is designed. Finally, for the target trajectories of maneuvering and non-maneuvering, computer simulation results show that the optimal fusion algorithm tracks very well maneuvering target over a wide range of change of measurement noise and maneuvering, the algorithm has the good robust performance of the approach, and it is very potential in practical engineering system.*

Keywords: Data fusion, target tracking, multi-sensor, fuzzy.

1 Introduction

As the sophistication and complexity of environment increase, it is very difficult and expensive to develop a new family of sophisticated sensors (working properly at all weather condition and all communications bandwidth). A multi-sensor system can improve the quality of coverage and the tracking performance than a stand-alone sensor in all weather conditions.

The problem of multi-sensor tracking has received a great deal of attention in recent years [1,2]. In the system, fusion can be used to combine sensory information to extend true track coverage and reduce false track formation. The advantage of fusing data at the signal level is its ability to draw conclusions directly from the data.

There have been developed many fusion techniques in multi-sensor tracking applications in recent years. Many researchers have considered different techniques for solving the multi-sensor tracking problem [3,4,5]. The algorithm of interacting multiple models (IMM) was considered in [3]. The algorithm of neural networks-based tracking for the "current" model was studied in [4]. Many data fusion and tracking methods were summarized in [5]. These techniques include the statistical methods; Dempster-Shaffer theory and neural networks based nonlinear fusion approaches. The statistical methods have been shown to be the most robust and perform the best among the approaches mentioned above. The statistical methods are mostly evolved from the estimation theory. However, adding new sensor metric information into a multi-sensor fusion tracking process does not always improve performance and can sometimes produce poorer results. References [6] and [7] used examples to show that—in some instances and contrary to expectation—adding new information resulted in poorer rather than improved performance, even though the information itself was correct. Reference [8] discussed the latitude in selecting weights for fusion and compared the different variance effects. These fusion and tracking approaches all require that the covariance of the sensory information be known a priori and unchanged. In practice, this assumption usually does not realism and has always been violated. Thus, the aforementioned fusion and tracking methods cannot be applied in any rigorous fashion and may be do real damage in systems.

2 Structure of optimal fusion system

In this paper, an on-line adaptive optimal fusion algorithm for multi-sensor tracking is presented. The system structure is shown in Fig.1. The principle of system is as follows: Assume that there are n sensors to track same maneuvering target, the sensor's measurements are fused in fusion center, two Unscented Kalman filters are used in parallel. One filter has a larger system variance to deal with all possible target maneuvers, and the system variance of the other filter is controlled adaptively by the output of fuzzy logic. The fuzzy logic calculates the best tuning output in real time to adapt to different target maneuvers according to the fused state information $x(k \mid k)$. The algorithm consists of two steps: (i) optimal fusion algorithm using on-line adaptive fuzzy inference method; (ii) tracking maneuvering target algorithm using the "current" statistical model and Unscented Kalman filter method. In the optimal fusion parts, the key idea of algorithm is how to maintaining the optimal fusion performance at any case. In order to do this, a suitable method of estimation measurements noise variance is developed, the algorithm can adapt itself to the changes of sensor's measurement noise, and the estimation error is of least mean square at any time. In the tracking parts, the two filters have the same state variables, namely position, velocity and acceleration. When the two filters are used in parallel, the key problem is how to utilize their outputs. The solution is that the system variance of one filter is tuned by fusing all the state information of the two filters' outputs through the fuzzy logic. The algorithm is suitable for different maneuvers of the target, and a higher tracking precision can be obtained.

This paper is organized as follows. In Section 3, we discuss minimum variance fusion approach. On-line adaptive weighted fusion algorithm is also discussed in this section. In Section 4, we discuss the fuzzy logic-based fusion and parallel adaptive tracking algorithm. Finally, In Section 5, computer simulations are used to demonstrate the performance of the proposed fusion approach.

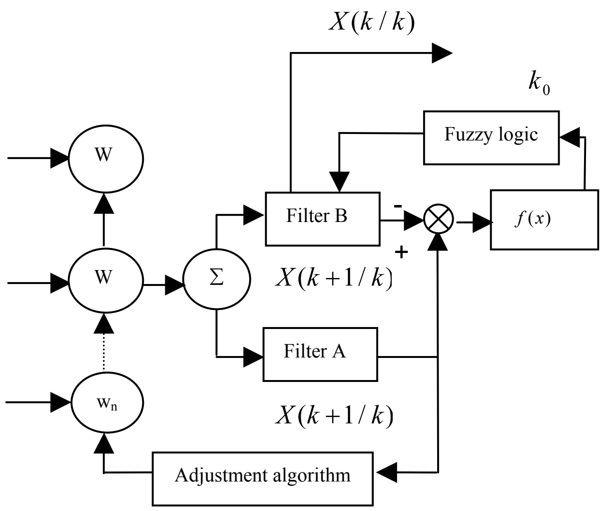

Fig.1 Principle of on-line optimal fusion tracking

3 Minimum Variance and optimal fusion

3.1 Minimum Variance fusion

Generally speaking fusion is to combine the sensory information in a systematic manner to get a good consensus of $X(t)$. Consider a system with n ($n \geq 2$) sensors. σ_n^2 is the measurement noise variance of the nth sensor. w_n denotes the weighting coefficient associated with the nth sensor. Using stochastic approximation theory, the minimum variance fusion weights can be obtained. Two theorems are given first.

Theorem 1: give matrix $A_i \in R^{l*l}$, $i = 1,2,...,r$, r is a positive matrix, and:

$$A_i = A_i^\tau, A_i > 0, \qquad i = 1,2,...,r.$$

$F(\bullet)$ is a nonlinear function, and : $\sum_{i=1}^{r} X_i = I_l$

The value of $\max F(\sum_{i=1}^{r} X_i A_i X_i^\tau)$ or $\min F(\sum_{i=1}^{r} X_i A_i X_i^\tau)$ is: $X_i^* = \left(\sum_{j=1}^{r} A_j^{-1}\right)^{-1} A_i^{-1}, \qquad i = 1,2,...,r.$

Proof: According to the method of Lagrange, the problem can be described as:

$$\partial\left(F\left(\sum_{j=1}^{r} X_j A_j X_j^\tau\right) + tr\lambda\left(\sum_{j=1}^{r} X_j - I_l\right)\right) \Big/ \partial X_i = 0, \; i = 1,2,...,r.$$

or:

$$\frac{\partial F\left(\sum_{j=1}^{r} X_j A_j X_j^\tau\right)}{\partial X_i} + \lambda^\tau = 0, \qquad i = 1,2,...,r.$$

Appendix 1 shows the full proof of the minimum variance fusion principle.

Theorem 2: given the sensor's measurement as follows:

$$x_{n+1}^i = y_n + a_n (h(y_n) + \varepsilon_{n+1}^i), \qquad 1 \leq i \leq r$$

$$y_n = \sum_{i=1}^{r} \lambda_n^i x_n^i, \qquad \sum_{i=1}^{r} \lambda_n^i = I, \qquad \forall n$$

Where y_n is fusion value, a_n is a step factor, $h(*)$ is a recursive function, ε_n is a measurement noise.

The variance of fusion is

$$S = \int_0^\infty e^{\left(H+\frac{1}{2}\alpha I\right)t} \left(\sigma + \sum_{i=1}^{r} \lambda^i R^i \lambda^{i\tau}\right) e^{\left(H+\frac{1}{2}\alpha I\right)^\tau t} dt$$

The optimal fusion weights is:

$$\lambda^{i*} = \left(\sum_{j=1}^{r} (R^j)^{-1}\right)^{-1} \left(R^i\right)^{-1} \qquad i \leq r$$

Proof: In order to obtain the optimal weight, the problem can be described as:

$$\lambda^* = \arg\min_{\lambda} \left(tr(S)\right)$$

Which is under the condition of:

$$\sum_{i=1}^{r} \lambda^{i*} = I$$

According to the theorem 1, it is a problem of optimal maximum or minimum of nonlinear matrix function, the optimal weight is:

$$\lambda^{i*} = \left(\sum_{j=1}^{r} (R^j)^{-1}\right)^{-1} \left(R^i\right)^{-1} \qquad i \leq r$$

According to the theorem 1 and theorem 2, the optimal weighting coefficients can be obtained by minimizing the variance of the fused information. Eq.1 and Eq.2 give the optimal weighting coefficient and corresponding optimal minimum variance, respectively:

$$w_p^* = 1 \Big/ \left|\sigma_p^2 \sum_{i=1}^{n} \frac{1}{\sigma_i^2}\right| \tag{1}$$

$$\sigma_{\min}^2 = 1 \Big/ \sum_{p=1}^{n} \frac{1}{\sigma_p^2} \tag{2}$$

If: $\sigma_1^2 = \sigma_2^2 = ... = \sigma_n^2 = \sigma^2$

Then:

$$\sigma_{\min}^{2} = \frac{\sigma^2}{n} \quad (3)$$

If the maximum standard deviation and minimum standard deviation of n ($n \geq 2$) sensors are:

$\sigma_{k\max}$ and $\sigma_{k\min}$

then, the optimal minimum variance is:

$$\sigma_y^2 = \frac{1}{\frac{1}{\sigma_{k\min}^2} + \frac{1}{\sigma_{k\max}^2} + \sum_{i=1}^{n-2} \frac{1}{\sigma_i^2}} \leq \frac{1}{\frac{1}{\sigma_{k\max}^2} + \sum_{i=1}^{n-2} \frac{1}{\sigma_i^2}} \quad (4)$$

From the Eq.3 and Eq.4, it can be seen that: in the general fusion case, optimal fusion will always result in an estimate that has better quality than either of the input estimates, even if the sensor data statistically have much poorer quality than the existing estimate. Conversely selecting different weights for the fusion process will produce a fusion result that is poorer than optimal and possibly poorer than either input.

Note that the optimal weighting coefficient relies on the exact a priori information about the sensor noise covariance. The minimum variance approach assumes that the sensor noise covariance is known. Although, this information can by obtained by some empirical techniques in practice, it is only an approximate of the true world and usually leads to a severely deteriorated fusion performance when the noise variance has changed. A conceivable idea is that the optimal fusion be obtained at any time if measurement noise variance can be estimated on-line.

3.2 Optimal fusion algorithm

According to the idea, an on-line adaptive weighted fusion approach was proposed to overcome above problem by estimating the variance of the fused information using statistical approximation theory.

Consider a system with n ($n \geq 2$) sensors. The fused measurement model of system can be written as:

$$X(t) = \sum_{i=1}^{n} w^i x^i(t) \qquad 1 \leq i \leq n \quad (5)$$

where:

$$\sum_{i=1}^{n} w^i = 1 \qquad 1 \leq i \leq n \quad (6)$$

In order to get the optimal weighting coefficient w^i, it is necessary to estimating variance of the fused information. Assume that measurement error can be described as:

$$\varepsilon^i = e^i + \xi^i \qquad i \leq n \quad (7)$$

$$E(e^i) = E(\xi^i) = 0 \qquad i \leq n \quad (8)$$

$$E(e^i * e^i) = \sigma, \quad E(\xi^i * \xi^i) = r \quad i \leq n \quad (9)$$

$$E(e^i * \xi^i) = 0 \qquad i \leq n \quad (10)$$

Given the function:

$$\psi_{k+1} = (1 - \tfrac{1}{k-1})\psi_k + \tfrac{1}{k-1}(x_k * x_k) \quad (11)$$

According to Eq.11, It can be proved that:

$$\psi_k \xrightarrow[k\to\infty]{} \sigma$$

So the value of estimating variance of the fused information can be computed as follows:

Assuming that the noise covariance matrix Q_k is known, here the method based on the technique known as covariance matching [9] has been derived to dynamically adjust the covariance matrix R_k.

The innovation sequence of Kalman Filter algorithm is:

$$d_k = \left(z_k - H_k \hat{x}_k^-\right) \quad (12)$$

Where z_k is the measurement value.

$$H_k P_k^- H_k^\tau + R_k = S_k \quad (13)$$

The actual covariance is approximated through averaging inside a moving estimation window of size M.

$$Cov\left(d_k\right) = \tfrac{1}{M} \sum_{i=i_0}^{k} d_i d_i^\tau \quad (14)$$

Where $i_0 = k - M + 1$ is the first sample inside the estimation window. This means that only the last M samples of d_k are used to estimate its covariance. In our algorithm, if it is found that the actual value of the covariance of d_k has no discrepancy with its theoretical value, then fuzzy inference method derives adjustments for R_k based on the knowledge of the size of this discrepancy. In order to detect the size of the discrepancy between S_k and $Cov(d_k)$, the degree of matching is defined as:

$$BeL_k = S_k - Cov\left(d_k\right) \quad (15)$$

It can be noted from (13) that an increment in R_k will increment S_k, and vice versa. This means that R_k can be used to vary S_k in accordance with the value of BeL_k in order to reduce the discrepancies between S_k and $Cov(d_k)$, from here three general rules of adjustment are defined as:

1) If $BeL_k \cong 0$ (this means S_k and $Cov(d_k)$ match almost perfectly) then maintain R_k unchanged.

2) If $BeL_k > 0$ (this means S_k is smaller than its actual value $Cov(d_k)$) then decrease R_k.

3) If $BeL_k < 0$ (this means S_k is greater than its actual value $Cov(d_k)$) then increase R_k.

Considering nine fuzzy sets for BeL_k, and seven fuzzy set for ΔR_k, using the compositional rule of inference sum prod and the center of area defuzzification method, R_k is adjusted in each step as given in (16).

$$R_k(i,i) = R_{k-1}(i,i) + \Delta R_k \quad (16)$$

Let $\theta^i(k)$ be the target measurement in spherical coordinates of sensor i,

$$\theta^i = [r^i \quad \eta^i \quad \varphi^i]' \quad (17)$$

Where r^i η^i φ^i are the measured range, bearing and elevation of sensor i respectively. The on-line adaptive fusion approach can be summarized as follows:

1) Compute the value of BeL_k, R_k according (15) and (16)
2) Obtain the variance of σ_r^2 σ_η^2 σ_φ^2 according (16).
3) Once the variance is obtained, the optimal weighting coefficient can be obtained according (1).
4) Finally, the estimating value of target movement can be calculated according minimum fusion approach.

4 Fuzzy logic-based fusion and parallel adaptive tracking algorithm

For the problem of tracking maneuvering targets, Professor Jing [4] proposed neural network-based information fusion and parallel adaptive tracking algorithm. The character of target motion model was described.

$$c_1 = f_1(X) = \frac{[\hat{x}_1(k+1/k) - \hat{x}_2(k+1/k)]^2}{s_1(k+1) + s_2(k+1)} \in [0.05, \quad 0.2]$$

$$c_2 = f_2(X) = \frac{[\hat{\dot{x}}_1(k+1/k) - \hat{\dot{x}}_2(k+1/k)]^2}{p_{1,22}(k+1) + p_{2,22}(k+1)} \in [0.2, \quad 0.6]$$

$$c_3 = f_3(X) = \frac{[\hat{\ddot{x}}_1(k+1/k) - \hat{\ddot{x}}_2(k+1/k)]^2}{p_{1,33}(k+1) + p_{2,33}(k+1)} \in [0.1, \quad 0.4]$$

Where:

$\hat{x}_i(k+1/k)$, $\hat{\dot{x}}_i(k+1/k)$, $\hat{\ddot{x}}_i(k+1/k)$ $(i = 1,2)$ are the predicting position, velocity and acceleration for the two filters respectively.

$s_1(k+1)$, $s_2(k+1)$ are the position error variance for the filter A and filter B.

$p_{i,22}(k+1/k)$, $p_{i,33}(k+1/k)$ $(i = 1,2)$ are the predicting variance of velocity and acceleration for two filters.

The concept of fuzzy sets and membership function can be used to adjust the output of c, the rule set c will be of the form antecedent →consequence:

R_i:

$IF(c_1 \text{ is } A_{i1})$ and $(c_2 \text{ is } A_{i2})$ and $(c_3 \text{ is } A_{i3})$

$THEN \quad k0 = K$

Where c_i is the vector of the target motion. A_i is the fuzzy set, the membership of input and output is in Fig.2 ,Fig.3 and Tab.1 respectively. The output of this fuzzy logic process can be described as follows:

$$k_0 = u^* = \frac{\sum_{i=1}^{m} \mu_i g_i}{\sum_{i=1}^{m} \mu_i} = \sum_{i=1}^{m} \overline{\mu_i} g_i \qquad (18)$$

where:

$$\mu_j = \prod A_{i1}(c_1) * A_{i2}(c_2) * A_{i3}(c_3)$$

$$\overline{\mu_i} = \frac{\mu_i}{\sum_{i=1}^{m} \mu_i}, \quad k_0 \in [0, \quad 1]$$

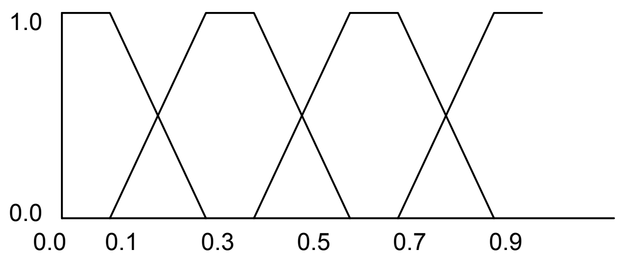

Fig.2 The membership function of input

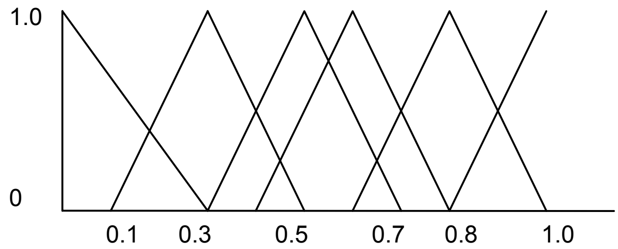

Fig.3 The membership function of output

TABLE 1
TABLE OF FUZZY OUTPUT

	E（ZE）	E（SP）	E（MP）	E（LP）
ΔE（ZE）	VP	SP	EP	EP
ΔE（SP）	LP	LP	VP	VP
ΔE（MP）	EP	VP	MP	MP
ΔE（LP）	VP	ZE	MP	EP

ZE is very small positive SP is small positive
MP is medium positive LP is large positive
VP is very large positive EP is small positive.

The target dynamics model in spherical coordinate is given by:

$$X(k+1) = \Phi(k+1,k)X(k) + \Gamma(k+1)\overline{\alpha} + s(k+1)$$

$$Y(k+1) = H[X(k+1)] + v(k+1)$$

where

$$X = [x, \ \dot{x}, \ \ddot{x}, \ y, \ \dot{y}, \ \ddot{y}, \ z, \ \dot{z}, \ \ddot{z}]'$$

$$Y = [r, \ \eta, \ \varphi]'$$

Because of the nonlinear in measurements equation, Unscented Kalman filter is used in tracking steps [10]. The tracking maneuvering target algorithm using the "current" statistical model and Unscented Kalman filter method is summarized as follow:

1) Compute the distribution of Sigma point:

$$\chi_k = [\hat{x}_k \quad \hat{x}_k + \sqrt{(n_x + \lambda)P_k} \quad \hat{x}_k - \sqrt{(n_x + \lambda)P_k}]$$

2) State update:

$$P_{k+1|k} = \sum_{i=0}^{2n_x} W_i^{(c)} (\chi_{i,k+1|k} - \hat{x}_{k+1|k})(\chi_{i,k+1|k} - \hat{x}_{k+1|k})^{\tau} + Q(k)$$

$$\hat{x}_{k+1|k} = \sum_{i=0}^{2n_x} W_i^{(m)} \chi_{i,k+1|k}$$
$$\chi_{k+1|k} = F\chi_k$$
$$\gamma_{k+1|k} = g(\chi_{k+1|k})$$
$$\hat{y}_{k+1|k} = \sum_{i=0}^{2n_x} W_i^{(m)} \gamma_{i,k+1|k}$$

3) Measurements update:

$$P_{\hat{y}_{k+1}\hat{y}_{k+1}} = \sum_{i=0}^{2n} W_i^{(c)} (\gamma_{i,k+1|k} - \hat{y}_{k+1|k})(\gamma_{i,k+1|k} - \hat{y}_{k+1|k})^{\tau} + R(k)$$
$$P_{x_{k+1}y_{k+1}} = \sum_{i=0}^{2n_x} W_i^{(c)} (\chi_{i,k+1|k} - \hat{x}_{k+1|k})(\gamma_{i,k+1|k} - \hat{y}_{k+1|k})^{\tau}$$
$$\kappa_{k+1} = P_{x_{k+1}y_{k+1}} P^{-1}_{\hat{y}_{k+1}\hat{y}_{k+1}}$$
$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + \kappa_{k+1}(z_{k+1} - \hat{y}_{k+1|k})$$
$$P_{k+1|k+1} = P_{k+1|k} - \kappa_{k+1} P_{\hat{y}_{k+1}\hat{y}_{k+1}} \kappa^{\tau}_{k+1}$$

Where:

$W_0^{(m)} = \lambda/(n_x + \lambda)$

$W_0^{(c)} = \lambda/(n_x + \lambda) + (1 - \alpha^2 + \beta)$

$W_i^{(m)} = W_i^{(c)} = 1/\{2(n_x + \lambda)\} \qquad i = 1, \cdots, 2n_x$

n_x is dimension of state, λ α β are the coefficient.

$Q_1(k) = 2\varsigma\sigma^2_{F_1} Q_0$, $Q_2(k) = 2\varsigma\sigma^2_{F_2} Q_0$

$$\sigma^2_{F_1} = \begin{cases} \dfrac{4-\pi}{\pi}(a_{\max} - \bar{a}_1)^2 & (a_1 \ge 0) \\ \dfrac{4-\pi}{\pi}(a_{-\max} - \bar{a}_1)^2 & (a_1 < 0) \end{cases}$$

$$\sigma^2_{F_2} = \begin{cases} k_0 * \dfrac{4-\pi}{\pi}(a_{\max} - \bar{a}_2)^2 & (a_2 \ge 0) \\ k_0 * \dfrac{4-\pi}{\pi}(a_{-\max} - \bar{a}_2)^2 & (a_2 < 0) \end{cases}$$

Where k_0 is the coefficient of fuzzy reasoning output, ς is the maneuvering frequency, Q_0 is a constant value matrix. α_1 and α_2 are the estimated values of current acceleration, respectively.

5 Numerical simulation studies

We use computation simulations to demonstrate the robustness performance of the proposed on-line optimal fusion algorithm. Two sensors are assumed, the sensor's measurements initial noises are assumed to be Gaussian process with zero mean value.

The tracking performance is chosen as:

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(x_{ij} - \hat{x}_{ij}^k)^2} \qquad (19)$$

The sampling interval is $T = 0.5\ s$. In constant velocity trajectory of simulation, $v = 150 m/s$, in maneuvering trajectory, the other parameters used are:

$\varsigma = 0.01$, $a_{\max} = 80 m/s^2$, $a = -10 m/s^2$, $a = 25 m/s^2$, the initial measurements noise covariance matrices for each sensor are as follows:

$$R_0^1 = \begin{bmatrix} 100^2 & 0 & 0 \\ 0 & 0.0175^2 & 0 \\ 0 & 0 & 0.0175^2 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} 100^2 & 0 & 0 \\ 0 & 0.0175^2 & 0 \\ 0 & 0 & 0.0175^2 \end{bmatrix}$$

Figure 4 and Figure 5 shows the measurements noise profiles for each sensor. Figure 6 and Figure7 gives the curve of the estimated weight of range. Figures 8 and Figure 9 shows the estimated velocity and acceleration error RMSE respectively for constant velocity trajectory. Figures 10 and Figure 11 shows the estimated velocity and acceleration error RMSE respectively for maneuvering trajectory. Table 1 gives the compare performance of the algorithm with the general method.

From the Fig.6~Fig.11, it can be seen that the general fusion method cannot eliminate the problem of time-vary measurements noise, fusion precision is decreased with time, the optimal fusion algorithm can eliminate the effects caused by the measurements noise. The on-line optimal fusion algorithm can adapt itself to the changes of sensor's noise, and the estimation error is of least mean square.

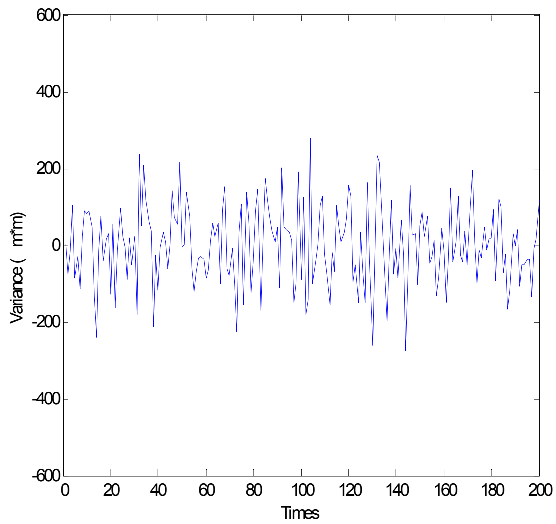

Fig.4 Constant Gaussian noise of sensor 1

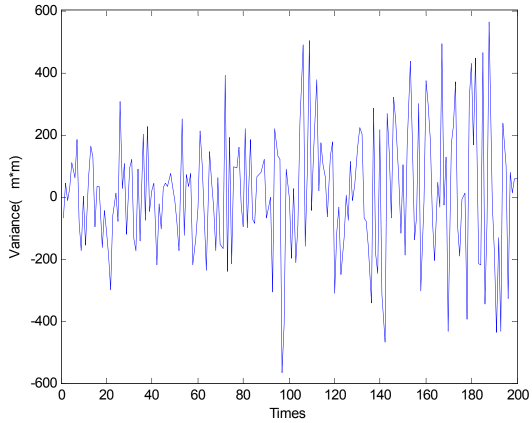

Fig.5 Time vary noise of sensor 2

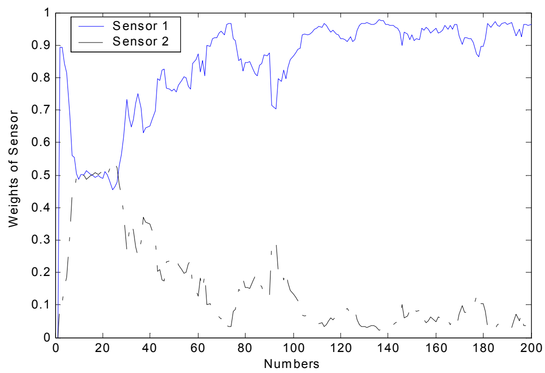

Fig.6 Estimated range weight of CV trajectory

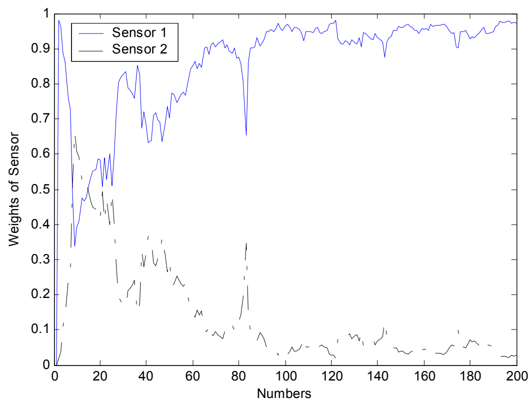

Fig.7 Estimated range weight of maneuvering trajectory

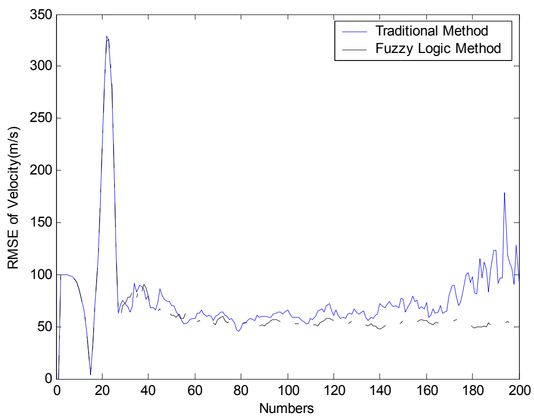

Fig.8 RMSE of velocity of CV trajectory

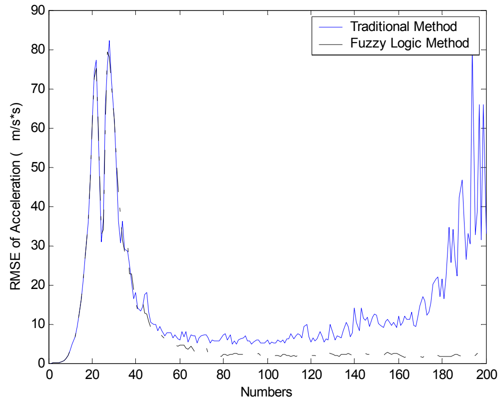

Fig.9 RMSE of acceleration of CV trajectory

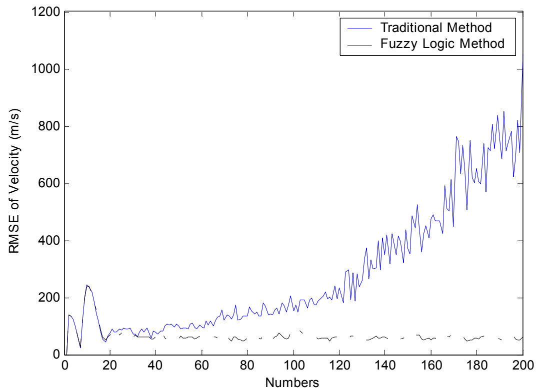

Fig.10 RMSE of velocity of maneuvering trajectory

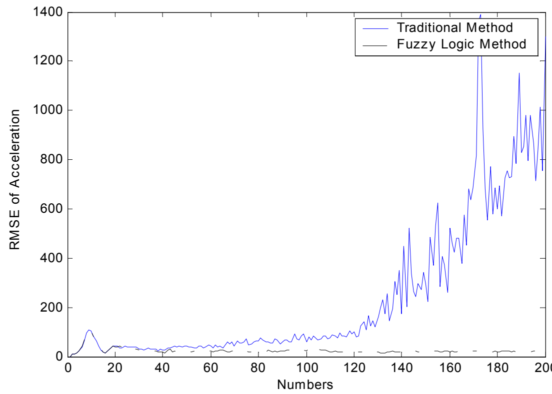

Fig.11 RMSE of acceleration of maneuvering trajectory

6 Conclusions

We propose a novel filtering and fuzzy logic architecture for tracking both maneuvering targets and time varying measurement noise processes. This algorithm is implemented with two filters and fuzzy logic using state fusion, together with the current statistic model and adaptive filtering. The proposed approach is able to handle the problems of unknown sensor noise variance. For the target trajectories of maneuvering and non-maneuvering, computer simulation results show that the optimal fusion algorithm tracks very well maneuvering target over a wide range of change of measurement noise and maneuvering, the algorithm has the good robust performance of the approach, and it has been shown to outperform the minimum variance in many situations, and it is very potential in practically useful for many real applications.

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APPENDIX I:

In order to solve the equation:

$$\frac{\partial F\left(\sum_{j=1}^{r} X_j A_j X_j^\tau\right)}{\partial X_i} + \lambda^\tau = 0, \qquad i = 1,2,\ldots,r.$$

Then:

$$\frac{\partial F\left(\sum_{j=1}^{r} X_j A_j X_j^\tau\right)}{\partial X_i} = \frac{\partial vec\left(X_i A_i X_i^\tau\right)^\tau}{\partial X_i} \left(I_l \otimes \left.\frac{\partial F(Z)}{\partial vecZ}\right|_{Z=\sum_{j=1}^{r} X_j A_j X_j^\tau} \right)$$

And:

$$\frac{\partial vec\left(X_i A_i X_i^\tau\right)^\tau}{\partial X_i} = \frac{\partial \left(vecA_i\right)^\tau \left(X_i^\tau \otimes X_i^\tau\right)}{\partial X_i} = \left(I_l \otimes \left(vecA_i\right)^\tau\right) \frac{\partial\left(X_i^\tau \otimes X_i^\tau\right)}{\partial X_i}$$

So, the equation can be changed as:

$\forall i = 1,2,\ldots,r$,

$$\sum_{i=1}^{r} X_i = I_l$$

and :

$$\lambda = -\left(I_l \otimes \left(vecA_i\right)^\tau\right)\left(U \otimes X_i^{*\tau} + \left(I_l \otimes U\right)\left(U \otimes X_i^{+}\right)\right) \bullet \left(I_l \otimes U\right) \left(I_l \otimes \left.\frac{\partial F(Z)}{\partial vecZ}\right|_{Z=\sum_{j=1}^{r} X_j^* A_j X_j^{*\tau}} \right)$$

Since:

$$Z = \sum_{j=1}^{r} X_j^* A_j X_j^{*\tau} = \left(\sum_{j=1}^{r} A_j^{-1}\right)^{-1}$$

$\left(I_l \otimes \left.\frac{\partial F(Z)}{\partial vecZ}\right|_{Z=\sum_{j=1}^{r} X_j^* A_j X_j^{*\tau}} \right)$ is a constant matrix.

Similar, the first and second parts are constant matrix also.

It can be described:

$$X_i^* = \left(\sum_{j=1}^{r} A_j^{-1}\right)^{-1} A_i^{-1}, \qquad i = 1,2,\ldots,r.$$

End.